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Dualities in $D = 5$, $N = 2$ Supergravity, Black Hole Entropy, and AdS Central Charges[1]

Dietmar Klemm[2]

Università degli Studi di Milano, Dipartimento di Fisica,
Via Celoria 16, 20133 Milano, Italy

Abstract: The issue of microstate counting for general black holes in $D = 5$, $N = 2$ supergravity coupled to vector multiplets is discussed from various viewpoints. The statistical entropy is computed for the near-extremal case by using the central charge appearing in the asymptotic symmetry algebra of AdS_2. Furthermore, we show that the considered supergravity theory enjoys a duality invariance which connects electrically charged black holes and magnetically charged black strings. The near-horizon geometry of the latter turns out to be $AdS_3 \times S^2$, which allows a microscopic calculation of their entropy using the Brown-Henneaux central charges in Cardy's formula. In both approaches we find perfect agreement between statistical and thermodynamical entropy.

1 Introduction

The study of black hole solutions in $N = 2$ five-dimensional supergravity coupled to vector and hypermultiplets plays an important role in the understanding of the non-perturbative structure of string and M-theory [1, 2]. In this setting the interplay between classical and quantum results is exemplified at its best.

In this paper we consider general charged black holes of the $D = 5$, $N = 2$ theories, not necessarily those obtained from compactification of eleven-dimensional supergravity on a Calabi-Yau threefold. The analysis is simplified by the rich geometric structure of the $N = 2$ theories. Black hole solutions are given in terms of a rescaled cubic homogeneous prepotential which defines very special geometry [3]. In the extremal BPS case, half of the vacuum supersymmetries are preserved, while at the horizon supersymmetry is fully restored [4].

Here we focus on the asymptotic symmetries of the near-horizon geometry of the general near-extremal solution: the aim is the computation of the entropy from a counting of microstates to be compared to the macroscopic, thermodynamical entropy.

We will see that the calculation of the microscopic entropy of small excitations above extremality is equivalent to a microstate counting for certain black holes in two-dimensional

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[2] E-mail: dietmar.klemm@mi.infn.it

anti-de Sitter space. This can then be done by using the central charge of the AdS_2 asymptotic symmetry algebra in Cardy's formula.

The main result however presented here is an explicit duality transformation, which realizes an invariance of the $N = 2$ supergravity action [5]. This duality turns the $AdS_2 \times S^3$ near-horizon geometry of the extremal black hole solution into $AdS_3 \times S^2$. The key point underlying the duality is the fact that the three-sphere can be written as a Hopf fibration over the base S^2. For AdS_3 the counting of microstates is performed using the Brown-Henneaux central charges [6] in Cardy's formula, and it is shown that this reproduces correctly the Bekenstein-Hawking entropy.

In the case where the $D = 5$, $N = 2$ supergravity action is obtained by Calabi-Yau (CY) compactification of M-theory, the considered duality transformation, which maps electrically charged black holes onto magnetically charged black strings, corresponds to the duality between M2 branes wrapping CY two-cycles and M5 branes wrapping CY four-cycles. According to [7], M-theory compactified on $AdS_3 \times S^2 \times M$, where M denotes some Calabi-Yau threefold, is dual to a $(0, 4)$ superconformal field theory living on an M5 brane wrapping some holomorphic CY four-cycle. This fact has been used in [8] to compute the entropy of five-dimensional BPS black holes[3]. We stress that our method for microstate counting applies to any near-extremal black hole in $N = 2$, $D = 5$ supergravity, independent of whether it is obtained by CY compactification or not.

In section 2, the black hole solutions of $N = 2$, $D = 5$ supergravity coupled to vector multiplets are briefly reviewed. We thereby focus on the STU model as a simple example, which nonetheless retains all the interesting features of the general solutions. In section 3 we compute the statistical entropy of small excitations near extremality, using the AdS_2 central charge [10], and find perfect agreement with the Bekenstein-Hawking entropy. In section 4 we construct the duality transformation for the supergravity action, and in 5 we finally perform the state counting, using the fact that the near-horizon geometry of the dual solution includes an AdS_3 factor. In this way, we obtain a microscopic entropy which agrees precisely with the corresponding thermodynamical result.

2 Black Holes in $N = 2$, $D = 5$ Supergravity

$N = 2$, $D = 5$ supergravity coupled to an arbitrary number n of Maxwell supermultiplets was first considered in [11]. In this theory, the scalar manifold can be regarded as a hypersurface in an $(n + 1)$-dimensional Riemannian space $\mathcal{R}$ with coordinates X^I. The equation of the hypersurface is $\mathcal{V} = 1$ where $\mathcal{V}$, the prepotential, is a homogeneous cubic polynomial in the coordinates of $\mathcal{R}$, $\mathcal{V}(X) = \frac{1}{6} C_{IJK} X^I X^J X^K$. One can then parametrize the hypersurface in terms of the n scalar fields ϕ^i appearing in the vector multiplets, $X^I = X^I(\phi^i)$.

The bosonic part of the Lagrangian is given by

$$e^{-1}\mathcal{L} = \frac{1}{2}R - \frac{1}{4}G_{IJ}F_{\mu\nu}{}^I F^{\mu\nu J} - \frac{1}{2}\mathcal{G}_{ij}\partial_\mu\phi^i\partial^\mu\phi^j + \frac{e^{-1}}{48}\epsilon^{\mu\nu\rho\sigma\lambda}C_{IJK}F^I_{\mu\nu}F^J_{\rho\sigma}A^K_\lambda . \tag{1}$$

The vector and scalar metric are completely encoded in the function $\mathcal{V}(X)$,

$$G_{IJ} = -\frac{1}{2}\partial_I\partial_J \ln \mathcal{V}(X)|_{\mathcal{V}=1}, \qquad \mathcal{G}_{ij} = G_{IJ}\partial_i X^I \partial_j X^J|_{\mathcal{V}=1}, \tag{2}$$

[3] The work in [8] includes as a special case also the results obtained in [9].

where ∂_i and ∂_I refer, respectively, to partial derivatives with respect to the scalar fields ϕ^i and X^I. Note that for Calabi-Yau compactifications of M-theory, C_{IJK} denote the topological intersection numbers, $\mathcal{V}(X)$ represents the intersection form, and X^I and $X_I = \frac{1}{6}C_{IJK}X^J X^K$ correspond, respectively, to the size of the two- and four-cycles of the Calabi-Yau threefold. In what follows, we will concentrate on the STU model [1, 12], i. e. $X^0 \equiv S$, $X^1 \equiv T$, $X^2 \equiv U$, $\mathcal{V}(X) = STU$. This model can be obtained by compactification of heterotic string theory on $K_3 \times S^1$ [13].

The field equations following from the action (1) admit the non-extremal static black hole solutions [14]

$$\begin{aligned} ds^2 &= -e^{-4V} f dt^2 + e^{2V}(f^{-1}dr^2 + r^2 d\Omega_3^2), \\ F^I_{rt} &= -H_I^{-2}\partial_r \tilde{H}_I, \qquad X^I = H_I^{-1}e^{2V}, \end{aligned} \tag{3}$$

where $d\Omega_3^2$ denotes the standard metric on the unit S^3. The H_I and $\tilde{H}_I$ are harmonic functions,

$$H_I = 1 + \frac{Q_I}{r^2}, \qquad \tilde{H}_I = 1 + \frac{\tilde{Q}_I}{r^2}, \tag{4}$$

where the $\tilde{Q}_I$ denote the physical electric charges. V and f read

$$e^{2V} = (H_0 H_1 H_2)^{1/3}, \qquad f = 1 - \frac{\mu}{r^2}, \tag{5}$$

with the nonextremality parameter μ. The physical charges are related to the Q_I by the equations

$$Q_I = \frac{\mu}{2}\sinh\beta_I \tanh\frac{\beta_I}{2}, \qquad \tilde{Q}_I = \frac{\mu}{2}\sinh\beta_I. \tag{6}$$

The extremal (BPS) limit is reached when $\beta_I \to \infty$, $\mu \to 0$, with $\mu \sinh\beta_I$ kept fixed.

For the ADM mass M_{ADM}, the Hawking temperature T_H, and the Bekenstein-Hawking entropy S_{BH}, one obtains

$$M_{ADM} = \frac{\pi}{4G_5}(\sum_I Q_I + \frac{3}{2}\mu), \qquad T_H = \frac{\mu}{\pi \prod_I (\mu + Q_I)^{1/2}}, \tag{7}$$

$$S_{BH} = \frac{A_{hor}}{4G_5} = \frac{\pi^2}{2G_5}\prod_I (\mu + Q_I)^{1/2}. \tag{8}$$

In the extremal case, the near-horizon geometry becomes $AdS_2 \times S^3$.

3 Statistical Entropy from AdS_2 Central Charge

We would now like to use the near-horizon geometry $AdS_2 \times S^3$ to count the microstates which give rise to the black hole entropy (8). As we are mainly interested in the AdS_2 factor, we perform a Kaluza-Klein reduction of the $D = 5$, $N = 2$ supergravity action (1) to two dimensions. As we only consider nonrotating black holes carrying electric charge, we can consistently truncate the Chern-Simons term in (1). The reduction ansatz for the metric is

$$ds^2 = \Phi^{-\frac{2}{3}} ds_2^2 + l_P^2 \Phi^{\frac{2}{3}} d\Omega_3^2, \tag{9}$$

where Φ denotes the dilaton and l_P is the Planck length in five dimensions. In two dimensions, the field strenghts F^I are proportional to the volume form and hence they can be integrated out. In this way, one arrives at the two-dimensional action

$$I = \frac{\pi}{8} \int d^2x\sqrt{-g} \left[\Phi R + \frac{6}{l_P^2 \Phi^{1/3}} - \Phi \mathcal{G}_{ij} \partial_\alpha \phi^i \partial^\alpha \phi^j - \frac{G^{IJ} \tilde{Q}_I \tilde{Q}_J}{l_P^6 \Phi^{5/3}} \right]. \tag{10}$$

Let us now expand the nonextremal black hole solution (3) near extremality. To this end, we introduce an expansion parameter ϵ ($\epsilon \to 0$), and set

$$\begin{aligned} t &= \frac{\tilde{t}}{\epsilon}, \qquad r = \sqrt{\frac{2l_P^2 \epsilon x}{(\tilde{Q}_0 \tilde{Q}_1 \tilde{Q}_2)^{(1/6)}} + \frac{\mu}{2}}, \qquad \mu = \mu_0 \epsilon, \\ \Phi &= \frac{(\tilde{Q}_0 \tilde{Q}_1 \tilde{Q}_2)^{1/2}}{l_P^3} + \frac{4}{\pi}\eta \qquad (\eta = \mathcal{O}(\epsilon)), \qquad \phi^i = \tilde{Q}_i^{-1} (\tilde{Q}_0 \tilde{Q}_1 \tilde{Q}_2)^{1/3} + \epsilon \tilde{\phi}^i. \end{aligned} \tag{11}$$

One thus arrives at

$$ds^2 = -(\lambda^2 x^2 - a^2) d\tilde{t}^2 + (\lambda^2 x^2 - a^2)^{-1} dx^2 \tag{12}$$

for the two-dimensional metric, with λ and a given by

$$\lambda = \frac{2l_P}{(\tilde{Q}_0 \tilde{Q}_1 \tilde{Q}_2)^{1/3}}, \qquad a^2 = \frac{\mu_0^2}{4l_P^2 (\tilde{Q}_0 \tilde{Q}_1 \tilde{Q}_2)^{1/3}}. \tag{13}$$

The action at lowest order in the expansion parameter ϵ reads

$$I = \frac{1}{2} \int d^2x \sqrt{-g}\eta [R + 2\lambda^2], \tag{14}$$

so the leading order is governed by the Jackiw-Teitelboim model. (12), together with the linear dilaton

$$\eta = \eta_0 \lambda x, \qquad \eta_0 = \frac{\Omega \epsilon}{16\pi l_P^2} (\tilde{Q}_0 \tilde{Q}_1 \tilde{Q}_2)^{2/3} \sum_I \tilde{Q}_I^{-1}, \tag{15}$$

represents a black hole solution of this model [10], with mass, thermodynamical entropy and temperature given by

$$M_{(2)} = \frac{1}{2}\eta_0 a^2 \lambda, \qquad S_{(2)} = 2\pi \eta_{hor} = 2\pi \eta_0 a, \qquad T_{(2)} = \frac{a\lambda}{2\pi}. \tag{16}$$

This black hole spacetime has constant curvature, i. e. it is locally AdS_2. Now it is known that the asymptotic symmetries of two-dimensional anti-de Sitter space form a Virasoro algebra [10], similar to the case of AdS_3, where one has two copies of Virasoro algebras as asymptotic symmetries [6]. This algebra was shown to have a central charge $c = 12\eta_0$ [15].

Expanding the ADM mass M_{ADM} (7) and Bekenstein-Hawking entropy S_{BH} (8) of the black hole (3) in five dimensions for $\mu \to 0$, one obtains that small excitations above extremality have the energy and entropy

$$\Delta M_{ADM} = \frac{\pi \mu^2}{32 l_P^3} \sum_I \tilde{Q}_I^{-1}, \qquad \Delta S_{BH} = \frac{\pi^2 \mu}{8 l_P^3} (\tilde{Q}_0 \tilde{Q}_1 \tilde{Q}_2)^{1/2} \sum_I \tilde{Q}_I^{-1}. \tag{17}$$

Comparing this with the two-dimensional results (16), one finds $\Delta S_{BH} = S_{(2)}$ and $\Delta M_{ADM} = \epsilon M_{(2)}$. The factor ϵ appearing in the relation between the two masses stems from the fact that M_{ADM} was computed with respect to the Killing vector ∂_t, whereas $M_{(2)}$ is related to $\partial_{\tilde{t}} = \epsilon \partial_t$. This means that up to these normalizations the five- and two-dimensional energies and entropies match.

Let us now compute the statistical entropy. Inserting the conformal weight $L_0 = M_{(2)}/\lambda$ together with the central charge in Cardy's formula $S_{stat} = 2\pi\sqrt{cL_0/6}$ yields a statistical entropy which agrees precisely with the thermodynamical entropy ΔS_{BH} of the small excitations above extremality.

4 Duality Invariance of the Supergravity Action

In this section we will show that in presence of a Killing vector field ∂_z, the supergravity action (1) is invariant under a certain generalization of T-duality[4]. The key observation is then that the three sphere S^3 appearing in the black hole geometry can be written as a Hopf fibration, i. e. as an S^1 bundle over $\mathbb{C}P^1 \approx S^2$. Performing then a duality transformation along the Hopf fibre untwists the S^3, and transforms the electrically charged black hole into a magnetically charged black string, which has $AdS_3 \times S^2$ as near-horizon limit in the extremal case.

To begin with, we reduce the action (1) to four dimensions, using the usual Kaluza-Klein reduction ansatz for the five-dimensional metric,

$$ds^2 = e^{k/\sqrt{3}} ds_4^2 + e^{-2k/\sqrt{3}}(dz + \mathcal{A}_\alpha dx^\alpha)^2, \tag{18}$$

where k denotes the dilaton, and early greek indices $\alpha, \beta, \ldots$ refer to four-dimensional spacetime. One thus arrives at the four-dimensional action

$$I_4 = \frac{L}{16\pi G_5}\int d^4x\sqrt{-g_4}\left[R_4 - \frac{1}{2}(\nabla k)^2 - \frac{1}{4}e^{-\sqrt{3}k}\mathcal{F}^2 - \frac{1}{2}e^{-k/\sqrt{3}}F^2 - \mathcal{G}_{ij}\partial_\alpha\phi^i\partial^\alpha\phi^j\right], \tag{19}$$

where L denotes the length of the circle parametrized by z, $\mathcal{F}$ is the field strength associated to the Kaluza-Klein vector potential $\mathcal{A}$, and

$$\mathcal{F}^2 = \mathcal{F}_{\alpha\beta}\mathcal{F}^{\alpha\beta}, \qquad F^2 = G_{IJ}F^I_{\alpha\beta}F^{J\alpha\beta}. \tag{20}$$

We now dualize both $\mathcal{F}$ and F^I, which yields

$$\begin{aligned} I_4 &= \frac{L}{16\pi G_5}\int d^4x\sqrt{-g_4}[R_4 - \frac{1}{2}(\nabla k)^2 - \frac{1}{4}e^{\sqrt{3}k}({}^\star\mathcal{F})^2 \\ &\quad -\frac{1}{2}e^{k/\sqrt{3}}\frac{1}{4}G^{IJ}\,{}^\star F_{I\alpha\beta}\,{}^\star F_J^{\alpha\beta} - \mathcal{G}_{ij}\partial_\alpha\phi^i\partial^\alpha\phi^j], \end{aligned} \tag{21}$$

where we defined

$${}^\star\mathcal{F}_{\alpha\beta} = \frac{1}{2}e^{-\sqrt{3}k}\epsilon_{\alpha\beta\gamma\delta}\mathcal{F}^{\gamma\delta}, \qquad {}^\star F_{I\alpha\beta} = e^{-k/\sqrt{3}}G_{IJ}\epsilon_{\alpha\beta\gamma\delta}F^{J\gamma\delta}. \tag{22}$$

Comparing (21) with (19), we observe that the gravitational and gauge field parts of the four-dimensional action, as well as the dilaton kinetic energy, are invariant under the $\mathbb{Z}_4$ transformation

$$k \to -k, \qquad \mathcal{F}_{\alpha\beta} \to {}^\star\mathcal{F}_{\alpha\beta}, \qquad F^I_{\alpha\beta} \to {}^\star F_{I\alpha\beta}, \qquad G_{IJ} \to \frac{1}{4}G^{IJ}. \tag{23}$$

[4] In what follows, we consistently truncate the Chern-Simons term. One can easily generalize the discussion below to nonvanishing CS term. This results in a θ term in four dimensions, which does not spoil the considered duality invariance.

The $\mathbb{Z}_4$ is actually a subgroup of the usual symplectic $Sp(2m+2,\mathbb{R})$ duality group [16] of $D=4$, $N=2$ supergravity (coupled to m vector multiplets). Note that the transformation $G_{IJ} \to G^{IJ}/4$ means that

$$X^I \to 3X_I = \frac{1}{2}C_{IJK}X^J X^K, \qquad X_I \to \frac{1}{3}X^I, \tag{24}$$

so essentially the special coordinates go over into their duals. As (24) does not change the kinetic term of the scalar fields, (23), (24) represent in fact a duality invariance of the four-dimensional action (19). In the special case of the $STU=1$ model, (24) implies that the moduli ϕ^i go over into their inverse, $\phi^i \to 1/\phi^i$. We now wish to apply the duality (23), (24) to the black hole solution (3). To this end, we consider the S^3 as an S^1 bundle over S^2, and write for its metric

$$d\Omega_3^2 = \frac{1}{4}\left[d\vartheta^2 + \sin^2\vartheta d\varphi^2 + (d\zeta + \cos\vartheta d\varphi)^2\right], \tag{25}$$

where ζ $(0 \le \zeta \le 4\pi)$ parametrizes the S^1 fibre. Introducing the coordinate $z = \lambda\zeta$, where λ denotes an arbitrary length scale, one can write the 5d metric in the KK form (18), where

$$\begin{aligned} ds_4^2 &= \frac{re^V}{2\lambda}\left[-e^{-4V}fdt^2 + e^{2V}f^{-1}dr^2 + e^{2V}\frac{r^2}{4}(d\vartheta^2 + \sin^2\vartheta d\varphi^2)\right], \\ e^{-k/\sqrt{3}} &= \frac{re^V}{2\lambda}, \qquad \mathcal{A} = \lambda\cos\vartheta d\varphi. \end{aligned} \tag{26}$$

We now dualize in 4d according to (23), and then relift the solution to five dimensions. This yields the configuration

$$\begin{aligned} ds^2 &= e^{-2V}\left[\frac{\mu}{4\lambda^2}dt^2 + 2dzdt + \frac{4\lambda^2}{r^2}dz^2\right] + \frac{r^2}{4\lambda^2}e^{4V}\left[f^{-1}dr^2 + \frac{r^2}{4}d\Omega_2^2\right], \\ F^I_{\vartheta\varphi} &= \frac{\tilde{Q}_I}{4\lambda}\sin\vartheta, \qquad X^I = H_I e^{-2V}. \end{aligned} \tag{27}$$

One effect of the duality transformation is thus the untwisting of the Hopf fibration[5].

One can further simplify (27) by an $SL(2,\mathbb{R})$ transformation

$$\begin{pmatrix} t' \\ z' \end{pmatrix} = \begin{pmatrix} 0 & -\frac{2\lambda}{\sqrt{\mu}} \\ \frac{\sqrt{\mu}}{2\lambda} & \frac{2\lambda}{\sqrt{\mu}} \end{pmatrix}\begin{pmatrix} t \\ z \end{pmatrix}. \tag{28}$$

Introducing also the new radial coordinate $\rho = r^2/(4\lambda)$, we then get for the metric

$$ds^2 = e^{-2V}(-fdt'^2 + dz'^2) + e^{4V}(f^{-1}d\rho^2 + \rho^2 d\Omega_2^2). \tag{29}$$

(29), together with the gauge and scalar fields given in (27), represents a nonextremal generalization of the supersymmetric magnetic black string found in [4]. The duality (23) thus maps electrically charged black holes onto magnetically charged black strings.

[5] The fact that Hopf bundles can be untwisted by T-dualities was observed in [17]. The idea of untwisting and twisting fibres to relate strings and black holes, and thus to gain new insights into black hole microscopics, was also explored in [18].

5 Microstate Counting from AdS_3 Gravity

We now want to use the near-horizon geometry of the dual solution (29) to count the microstates giving rise to the Bekenstein-Hawking entropy. In [4] it was shown that in the extremal case, the geometry becomes $AdS_3 \times S^2$ near the event horizon. The idea is now to use the central charge of AdS_3 gravity [6] in Cardy's formula, in order to compute the statistical entropy, like it was done by Strominger [19] for the BTZ black hole[6]. As only the AdS_3 part is relevant, we would like to reduce the supergravity action from five to three dimensions. To this end, we first Hodge-dualize the magnetic two-form field strength in (27). For the solution under consideration, the field strengths H_I dual to the F^I do not depend on the coordinates of the internal S^2. Furthermore, in 3d the three-forms H_I are proportional to the volume form and can be integrated out. For the metric, we use the reduction ansatz

$$ds^2 = \Phi^{-1} ds_3^2 + l_P^2 \Phi^2 d\Omega_2^2, \tag{30}$$

where $d\Omega_2^2$ denotes the standard metric on the unit S^2. This gives the reduced action

$$I = \frac{1}{4l_P} \int d^3x \sqrt{-g}\, \Phi^{\frac{3}{2}} \left[R + \frac{2}{l_P^2 \Phi^3} - \frac{3}{2\Phi^2}(\nabla\Phi)^2 - \frac{G_{IJ} P^I P^J}{\Phi^5 l_P^4} - \mathcal{G}_{ij} \partial_\alpha \phi^i \partial^\alpha \phi^j \right], \tag{31}$$

where we introduced the magnetic charges $P^I = \tilde{Q}_I/(4\lambda)$ of the black string (27). The idea is now to expand the 3d metric ds_3^2 near the horizon and near extremality. This can be done by setting

$$t' = \frac{t''}{\sqrt{\epsilon}} (2\lambda)^4 \sqrt{\frac{l_P}{\mu_0 \lambda \tilde{Q}_0 \tilde{Q}_1 \tilde{Q}_2}}, \qquad z' = \frac{z''}{\sqrt{\epsilon}} \frac{(2\lambda)^2}{\sqrt{\mu_0}}, \qquad \rho = \epsilon \tilde{r}^2 \frac{\mu_0 l_P}{(2\lambda)^4}, \qquad \mu = \mu_0 \epsilon, \tag{32}$$

and taking the limit $\epsilon \to 0$. This leads to the metric

$$ds_3^2 = -\frac{\tilde{r}^2 - \tilde{r}_+^2}{l_{eff}^2} dt''^2 + \tilde{r}^2 dz''^2 + \frac{l_{eff}^2 d\tilde{r}^2}{\tilde{r}^2 - \tilde{r}_+^2}, \tag{33}$$

where we introduced

$$\tilde{r}_+^2 = \frac{4\lambda^3}{l_P}, \qquad l_{eff}^2 = \frac{\tilde{Q}_0 \tilde{Q}_1 \tilde{Q}_2}{16 l_P \lambda^3}. \tag{34}$$

We recognize (33) as the BTZ black hole, with event horizon at $\tilde{r} = \tilde{r}_+$. $\Lambda_{eff} = -1/l_{eff}^2$ is the effective cosmological constant. The effective 3d Newton constant can be read off from the action (31), yielding

$$\frac{1}{16\pi G_{eff}} = \frac{1}{4l_P} \Phi_{hor}^{3/2}, \tag{35}$$

where the subscript indicates that the dilaton Φ is to be evaluated at the horizon. The BTZ black hole mass is given by

$$M_{(3)} = \frac{\lambda^3}{2 l_P G_{eff} l_{eff}^2}. \tag{36}$$

We can now apply Strominger's counting of microstates [19] to reproduce the Bekenstein-Hawking entropy. To this end, one first observes that the central charge appearing in the

[6] Cf. also [20], where similar computations for black strings in six dimensions with $BTZ \times S^3$ near-horizon geometry were performed.

asymptotic symmetry algebra of AdS_3 [6] in our case reads $c = 3l_{eff}/(2G_{eff})$. Furthermore, we have the relations

$$M_{(3)} = \frac{1}{l_{eff}}(L_0 + \bar{L}_0), \qquad J = L_0 - \bar{L}_0 = 0 \tag{37}$$

for the mass and angular momentum. We then obtain from Cardy's formula a statistical entropy which coincides precisely with the thermodynamical entropy (8) of the 5d black hole (3).

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References

[1] A. Chou, R. Kallosh, J. Rahmfeld, S. -J. Rey, M. Shmakova, and W. K. Wong, Nucl. Phys. **B508** (1997) 147.

[2] I. Gaida, S. Mahapatra, T. Mohaupt, and W. A. Sabra, Class. Quant. Grav. **16** (1999) 419.

[3] B. de Wit and A. Van Proeyen, Phys. Lett. **B293** (1992) 94.

[4] A. H. Chamseddine and W. A. Sabra, Phys. Lett. **B460** (1999) 63.

[5] S. Cacciatori, D. Klemm, W. A. Sabra, and D. Zanon, Nucl. Phys. **B587** (2000) 277.

[6] J. D. Brown and M. Henneaux, Commun. Math. Phys. **104** (1986) 207.

[7] J. M. Maldacena, Adv. Theor. Math. Phys. **2** (1998) 231.

[8] C. Vafa, Adv. Theor. Math. Phys. **2** (1998) 207.

[9] A. Strominger and C. Vafa, Phys. Lett. **B379** (1996) 99.

[10] M. Cadoni and S. Mignemi, Phys. Rev. **D59** (1999) 081501; Nucl. Phys. **B557** (1999) 165.

[11] M. Günaydin, G. Sierra, and P. K. Townsend, Nucl. Phys. **B242** (1984) 244.

[12] W. A. Sabra, Mod. Phys. Lett. **A13** (1998) 239.

[13] I. Antoniadis, S. Ferrara, and T. R. Taylor, Nucl. Phys. **B460** (1996) 489.

[14] K. Behrndt, M. Cvetič, and W. A. Sabra, Nucl. Phys. **B553** (1999) 317.

[15] M. Cadoni and M. Cavaglià, hep-th/0005179; hep-th/0008084; M. Caldarelli, G. Catelani, and L. Vanzo, JHEP **0010** (2000) 005; M. Cadoni, P. Carta, D. Klemm, and S. Mignemi, hep-th/0009185.

[16] B. Craps, F. Roose, W. Troost, and A. Van Proeyen, Nucl. Phys. **B503** (1997) 565.

[17] M. J. Duff, H. Lü, and C. N. Pope, Nucl. Phys. **B532** (1998) 181; Nucl. Phys. **B544** (1999) 145; E. Halyo, Mod. Phys. Lett. **A15** (2000) 397.

[18] M. Cvetič, H. Lü, and C. N. Pope, Nucl. Phys. **B549** (1999) 194.

[19] A. Strominger, JHEP **9802** (1998) 009.

[20] M. Cvetič and F. Larsen, Nucl. Phys. **B531** (1998) 239.